The Interaction Rate in Holographic Models of Dark Energy

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Abstract. Observational data from supernovae type Ia, baryon acoustic oscillations, gas mass fraction in galaxy clusters, and the growth factor are used to reconstruct the the interaction rate of the holographic dark energy model recently proposed by Zimdahl and Pavón [1] in the redshift interval $0 < z < 1.8$. It shows a reasonable behavior as it increases with expansion from a small or vanishing value in the far past and begins decreasing at recent times. This suggests that the equation of state parameter of dark energy does not cross the phantom divide line.

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INTRODUCTION

There is a wide-shared conviction among cosmologists that the Universe is nowadays experiencing a stage of accelerated expansion not compatible with the up to now favored Einstein-de Sitter model. This consensus, however, does not extend to the agent (very frequently called dark energy) behind this acceleration and, at the moment, there are many competing candidates [2], the cosmological constant being the leading one. However, the latter suffers from two main short-comings: the huge value predicted by quantum field theory estimations, and the coincidence problem, i.e., the fact that the energy densities of non-relativistic matter and dark energy are currently of the same order. This is why many researches are looking for alternative candidates of dark energy. Among the most recent generic proposals there is a very suggestive one based on the holographic principle. Loosely speaking, the latter asserts that the entropy of a system is given by the number of degrees of freedom lying on the surface that bounds it, rather than in its volume [3]. The roots of this principle are to be found in the thermodynamics of black holes [4]. Nevertheless, as observed by Cohen *et al.* [5], a system may fulfill the holographic principle and, however, include states for which its Schwarzschild radius is larger than its size, L. This can be avoided by imposing that the energy of the system should not exceed that of black hole of the same size or, equivalently, $\rho \leq 3c^2/(8\pi G L^2)$, where c^2 is a (non-necessarily constant) parameter. In the cosmological context L, the infrared cutoff, is usually taken either as the event horizon radius or the Hubble radius. For a quick summary of holographic dark energy see section 3 of Ref. [1].

The model of Zimdahl *et al.* [1] rests in two main assumptions: (i) dark energy complies with the holographic principle with L identified as the radius of the Hubble horizon, H^{-1}, hence $\rho_x = 3c^2H^2/(8\pi G)$, and (ii) dark energy and dark matter do not evolve separately but they interact with one another. As a consequence, the energy

conservation equations read

$$\dot{\rho}_m + 3H\rho_m = Q\,, \qquad \dot{\rho}_x + 3H(1+w)\rho_x = -Q\,, \tag{1}$$

where $w = p_x/\rho_x$ is the equation of state parameter of dark energy which is not constrained to be a constant. Subscripts m and x are for non-relativistic dark matter and dark energy, respectively.

It is to be noted that for spatially flat universes in the absence of interaction, $Q = 0$, there would be no acceleration [1, 6]. Moreover, Q must be a positive-definite quantity if the coincidence problem is to be solved [7] or at least alleviated [8], and the second law of thermodynamics to be fulfilled [9]. Further if Q were negative, ρ_x would have been negative in the far past. Besides, it has been forcefully argued that the Layzer-Irvine equation [10] when applied to galaxy clusters reveals the existence of the interaction [11]. To the best of our knowledge, the interaction hypothesis was first introduced, much earlier of the discovery of late acceleration, by Wetterich [12] to reduce the theoretical huge value of the cosmological constant, and was first used in the holography context by Horvat [13]. As we write, the body of literature on the subject is steadily growing -see [1, 14] and references therein. Most cosmological models implicitly assume that matter and dark energy couple gravitationally only. However, unless there exists an underlying symmetry that would set Q to zero (such a symmetry is still to be discovered) there is no *a priori* reason to discard the interaction. Ultimately, observation will tell us whether the interaction exists.

Following [1], we will write the interaction as $Q = \Gamma\rho_x$, where Γ is an unknown, semi-positive definite, function that gauges the rate at which energy is transferred from dark energy to dark matter. Clearly, as long as the nature of both dark ingredients of the cosmic substratum remain unknown, Γ cannot be derived from first principles. The alternative is to resort to observational data (in our case, supernovae type Ia (SN Ia), baryon acoustic oscillations (BAO), gas mass fraction in galaxy clusters and the growth factor) to roughly reconstruct it -for details see [15].

RECONSTRUCTING THE RATE

From Eqs. (1) and the above expressions for ρ_x and Q the evolution equation

$$\dot{r} = (1+r)\left[3Hw\frac{r}{1+r} + \Gamma\right]\,, \tag{2}$$

for the ratio $r \equiv \rho_m/\rho_x$ between the energy densities, is readily obtained. With the help of Friedmann equation $\Omega_m + \Omega_x + \Omega_k = 1$, in terms of the usual density parameters $\Omega_i = 8\pi G\rho_i/(3H^2)$ $(i = m, x)$, and $\Omega_k = -k/(a^2H^2)$, where k stands for the spatial curvature index of the Friedmann-Robertson-Walker metric, we can write

$$\dot{r} = -2H\,\frac{\Omega_k}{\Omega_x}\,q\,, \tag{3}$$

where $q = -\ddot{a}/(aH^2)$ denotes the cosmic deceleration parameter. Here, for the holographic dark energy we have adopted the expression $\rho_x \propto H^2$. The latter follows from choosing the infrared cutoff, L, as the Hubble radius, H^{-1}.

Likewise, starting from the first of Eqs. (1) and using Friedmann equation, we get for the equation of state parameter

$$w(z) = (1+r)\left[\frac{2}{3}\frac{H'}{H} - 1\right] - \frac{2}{3}\frac{\Omega_k}{\Omega_x}\left[1-(1+z)\frac{H'}{H}\right], \tag{4}$$

where z denotes the redshift and a prime indicates derivative with respect to this quantity.

We fit the Chevallier-Polarsky-Linder parametrization [16], namely,

$$w(z) = w_0 + w_1\frac{z}{1+z}, \tag{5}$$

where w_0 is the present value of $w(z)$, and w_1 a further constant, to current data from different observational probes and subsequently use the fitting values for w_0 and w_1 to reconstruct the dimensionless ratio $\Gamma/3H$.

As for the data, we resort to the various SN Ia observations in recent times. In particular we use 60 Essence supernovae [17], 57 SNLS (Supernova Legacy Survey) and 45 nearby supernovae. We have also included the new data release of 30 SNe Ia spotted by the Hubble Space Telescope and classified as the Gold sample by Riess *et al.* [17]. The combined data set can be found in Ref. [18]. The total number of data points involved is 192.

Next we add the measurement of the cosmic microwave background (CMB) acoustic scale at $z_{BAO} = 0.35$ as observed by the Sloan digital sky survey (SDSS) for the large scale structure. This is the BAO peak [19].

We also consider the gas mass fraction of galaxy cluster, $f_{gas} = M_{gas}/M_{tot}$, inferred from the x-ray observations [20]. This depends on the angular diameter distance d_A to the cluster as $f_{gas} = d_A^{3/2}$. The number of data points involved is 26.

Likewise, the two-degree field galaxy redshift survey (2dFGRS) has measured the two point correlation function at an effective redshift of $z_s = 0.15$. This correlation function is affected by systematic differences between redshift space and real space measurements due to the peculiar velocities of galaxies. Such distortions are expressed through the redshift distortion parameter, β. Correlation function can be used to measure it as $\beta = 0.49 \pm 0.09$ at the effective redshift of z = 0.15 of the 2dF survey. This result can be combined with linear bias parameter $b = 1.04 \pm 0.11$ obtained from the skewness induced in the bispectrum of the 2dFGRS by linear biasing to find the growth factor g at $z = 0.15$, namely $g = 0.51 \pm 0.11$ [21].

The spatially flat case

By setting $\Omega_k = 0$ equations (2) and (4) reduce to

$$\frac{\Gamma}{3H} = -r_0 \left[\frac{2}{3} \frac{H'}{H} - 1 \right] , \tag{6}$$

and

$$w(z) = (1 + r_0) \left[\frac{2}{3} \frac{H'}{H} - 1 \right] , \tag{7}$$

respectively[1]. As usual, r_0 indicates the present value of the ratio r. Using these two expressions we determine w_0 and w_1 from the data and, with them, we reconstruct $\Gamma/3H$ -see figures 1 and 2. The best fit values, with 1σ error bars for the parameters when all the data (SN Ia + BAO + x-rays + growth factor) are included, come to be: $w_0 = -1.13 \pm 0.24$, $w_1 = 0.66 \pm 1.35$ (for $\Omega_{m0} = 0.25$ & $\Omega_k = 0$, Fig. 1); and $w_0 = -0.80 \pm 0.28$, $w_1 = -1.75 \pm 1.79$ (for $\Omega_{m0} = 0.3$ & $\Omega_k = 0$, Fig. 2).

Contrary to what one may think, the fact that r was never large does not bring the model of Ref. [1] into conflict with the standard scenario of cosmic structure formation. One may believe that at early times the amount of dark matter would have been too short to produce gravitational potential wells deep enough for galaxies to condensate. However, this is not so; a matter dominated phase is naturally recovered since at high and moderate redshifts the interaction is even smaller than at present whence the equation of state of the dark energy becomes close to that of non-relativistic matter -see [1] for details.

Likewise, it should be noted that dark energy clusters similarly to dark matter when the equation of state of the former stays close to that of the latter. To see this more clearly we recall the perturbation dynamics of this model studied in [1] by using the perturbed metric $\mathrm{d}s^2 = -(1+2\psi)\,\mathrm{d}t^2 + a^2\,(1-2\psi)\,\delta_{\alpha\beta}\mathrm{d}x^\alpha \mathrm{d}x^\beta$ with ψ the scalar metric perturbation, and the Bardeen gauge-invariant variable [22]

$$\zeta \equiv -\psi + \frac{1}{3} \frac{\hat{\rho}}{\rho + p} = -\psi - H \frac{\hat{\rho}}{\dot{\rho}} \, . \tag{8}$$

The latter represents curvature perturbations on hypersurfaces of constant energy density. For the simplest case $\Gamma =$ constant, one follows

$$\zeta = \zeta_i - \frac{\Gamma}{3} \frac{\hat{r}}{r} \frac{1}{3H_i\, r - \Gamma} \left[\left(\frac{a}{a_i} \right)^{3/2} - 1 \right] , \tag{9}$$

where the subscript i signals some initial time and a hat indicates perturbation of the

[1] The corresponding equations (namely (6) & (7)) in [15] bear an extra factor, $(1+z)$. This was a typo with no further consequences to the calculations. This corrects the typo.

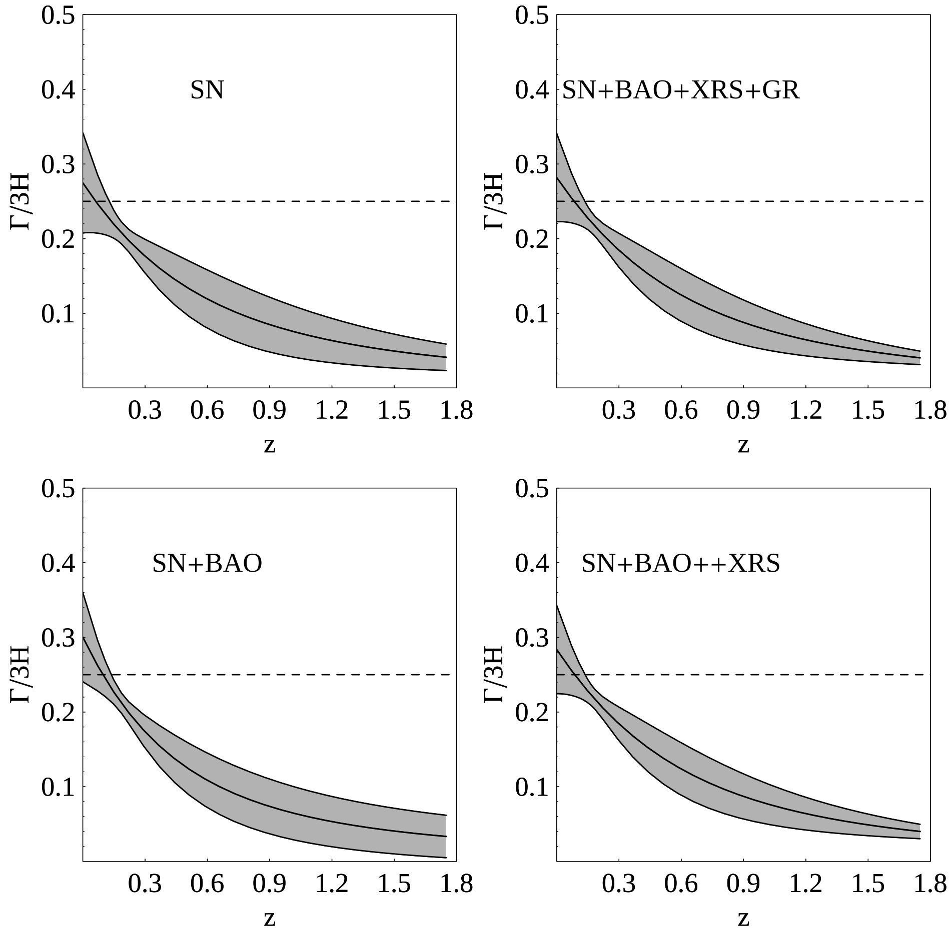

FIGURE 1. The dimensionless ratio $\Gamma/(3H)$ vs redshift. In the four panels we have fixed $\Omega_{m0} = 0.25$ and $\Omega_k = 0$. The solid line is for the mean value and the shaded area indicates the 1σ region. The region above the horizontal dashed line can be visited only when the dark energy becomes of phantom type, i.e., $w < -1$.

corresponding quantity -see [1] and [15] for details. Accordingly, as far as the the equation of state parameter of dark energy w remains close to that of dark matter, $w \simeq 0$, both components cluster similarly.

Non-spatially flat cases

It is immediately seen that for $\Omega_k \neq 0$ the ratio r between energy densities is not a constant. This introduces a further unknown function in our fitting procedure. Notwithstanding, one should not expect a large variation in r in the redshift range $(0, 1.8)$. In our subsequent computation, we Taylor expand r around its present value up to the first

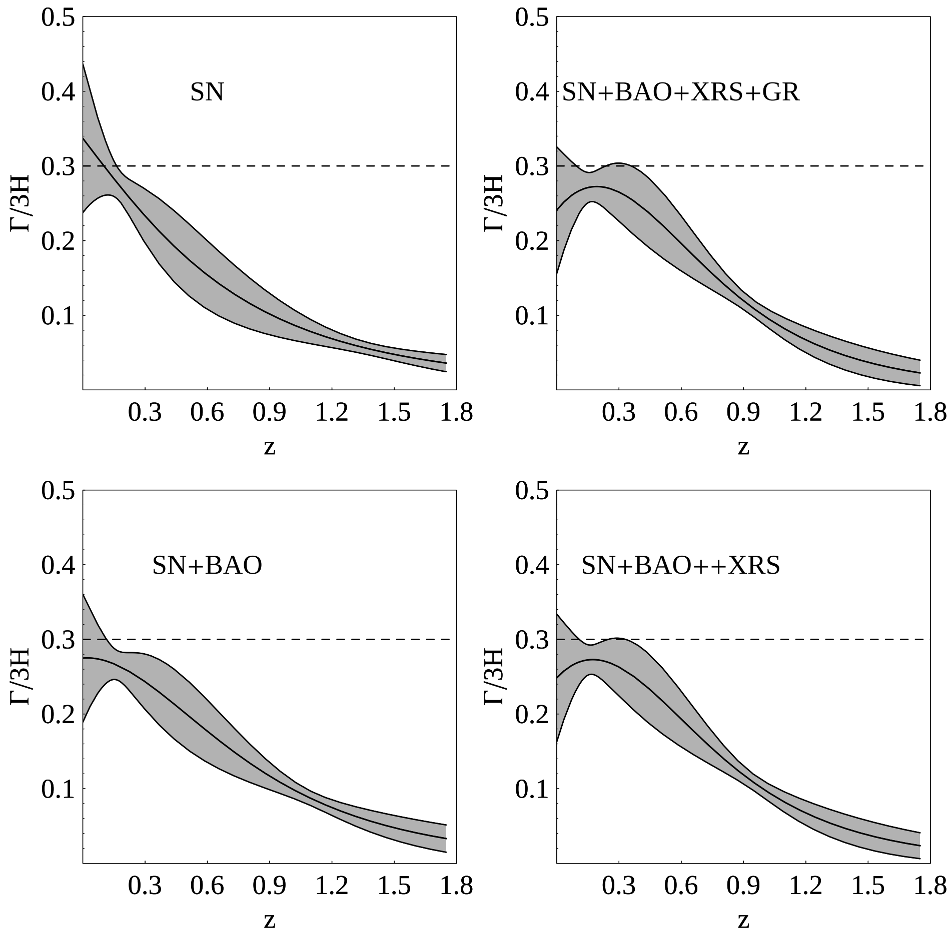

FIGURE 2. Same as Fig. 1 except that here we have fixed $\Omega_{m0} = 0.30$ and $\Omega_k = 0$.

order term. Accordingly we parameterize it as

$$r = r_0 + r_1 \frac{z}{1+z}. \tag{10}$$

Here r_1 is a constant which can be related to the present ratio between Ω_k and Ω_x by

$$\frac{\Omega_{k0}}{\Omega_{x0}} = -\frac{r_1}{2}\left[1 - \left(\frac{H'}{H}\right)_{z=0}\right]. \tag{11}$$

This can be used to fix the unknown constant r_1 for a given Ω_{k0} and Ω_{x0}. Also

$$\frac{\Gamma}{3H} = -\frac{1}{1+r}\left(r' \frac{1+z}{3} - wr\right), \tag{12}$$

where w is given by Eq. (4).

With the help of these expressions, the ratio $\Gamma/3H$ can be reconstructed from the data. The corresponding figure (with $\Omega_{m0} = 0.30$ and small Ω_{k0}) is very similar to Fig. 2 thereby we do not reproduce it here (see Fig. 3 in Ref. [15]). The best fit values, with 1σ error bars for the parameters when all the data (SN Ia + BAO + x-rays + growth factor) are included, come to be: $w_0 = -0.806 \pm 0.29$, $w_1 = -1.74 \pm 3.33$. We conclude that a small spatial curvature, in agreement with the WMAP 5yr experiment [23], has a negligible impact on the evolution of the interaction rate.

DISCUSSION

Using the observational data (SNIa, BAO, gas mass fraction, and growth factor) available in the redshift range $0 < z < 1.8$ we reconstructed the interaction term Q of Ref. [1]. The interaction rate Γ (and hence Q) stays positive in the said range. Its general trend is to decrease as z increases but it shows no indication of becoming negative at larger redshifts. This corroborates that, as previously suggested [9, 11], the energy transfer proceeds from dark energy to dark matter, not the other way around. Although phantom behavior cannot be excluded at recent and present times it only occurs manifestly either for large Ω_{x0} -see Fig. 1- or when just the supernovae data are considered (top-left panel of Figs. 1 and 2). When Ω_{x0} is somewhat lower (say, 0.7) and BAO and other data are included, the mean value of dimensionless interaction rate, $\Gamma/3H$, no longer crosses the phantom divide (i.e., the horizontal dashed line). It reaches a maximum near $z = 0$ to subsequently decrease with expansion. This result is rather comforting since holography does not seem compatible with phantom energy [24]. On the other hand, it should be noticed that Ω_{x0} values as high as 0.75 do not appear favored from a combination of results from WMAP 1yr and weak lensing which yields $\Omega_{x0} = 0.70 \pm 0.3$ [25].

Likewise, a small curvature term -of either sign- is of little consequence.

At any rate, it is fair to say that the concordance ΛCDM model ($w_0 = -1$, $w_1 = 0$) shows compatibility within 1σ confidence level with the set of data considered in this paper.

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